**Macroscopic laws as laws of large numbers**
**in non-ergodic Hamiltonian dynamics**

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**Abstract of the lectures**

**Lecture 1 : Introduction (Nov. 9)**

By considering the example of the ordinary diffusion equation, I will start by explaining why and how phenomenological laws of macroscopic physics should be understood as laws of large numbers. I will introduce the random Lorentz gas and the periodic Lorentz gas. In this context, I will recall the landmark result of Bunimovich and Sinai. Finally, I will introduce and review results about the lattice version of the random Lorentz gas : the mirrors model.

**Lecture 2 : The Kac's ring model. (Nov. 10)**

There are two classical objections to the derivation of irreversible macroscopic laws from microscopic dynamics. The Zermelo paradox argues that because of the Poincare recurrence theorem, basic ally any dynamical system in a finite domain should come back (close) to its initial state thereby contradicting the observed irreversible evolution of macroscopic quantities. The Loschmidt paradox uses the fact that if it is possible to find an initial state leading to the observed irreversible behaviour of a system, then, reversing all velocities of the particles at any stage of the evolution yields other initial conditions that lead to the opposite behaviour. The Kac ring model was devised by Mark Kac as an example of non-ergodic, periodic and reversible dynamics for which it is possible to derive rigorously an irreversible evolution of macroscopic quantities. I will give explain his model and a clear proof in details.

**Lecture 3 : The rings model. (Nov. 11)**

The Kac's ring model can not serve as a model for deriving the ordinary diffusion of particles or energy in spatially extended systems. I will introduce a model that is based on similar ideas than the Kac's ring model and fills this gap. As in the Kac's model, the dynamics of the model acts on a discrete phase space at discrete times but has nonetheless some of the characteristic properties of Hamiltonian dynamics in a confined phase space : it is deterministic, periodic, reversible and conservative. It is also non-ergodic in a non-obvious way. Randomness enters the model as a way to model ignorance about initial conditions and interactions between the components of the system. The orbits of the particles are nonintersecting random loops. We will prove, by a weak law of large number, the validity of a diffusion equation for the macroscopic observables of interest for times that are arbitrary large, but small compared to the minimal recurrence time of the dynamics

**Lecture 4 : Derivation of Fick's law for the current of particles in lattice random Lorentz gases out of equilibrium (Nov. 12)**

I will provide a (or sketch of) proof that the stationary macroscopic current of particles in a special type of random lattice Lorentz gas satisfies Fick's law when connected to particles reservoirs. We will consider a box on a d+1 dimensional lattice and when d is greater than or equal to 7, we will show that under a diffusive rescaling of space and time, the probability to find a current different from its stationary value is exponentially small in time. Its stationary value is given by the conductivity times the difference of chemical potentials of the reservoirs. The proof is based on the fact that in high dimension, random walks have a small probability of making loops or intersecting each other when starting sufficiently far apart.

**Lecture 5 : Perspectives on the general lattice random Lorentz gas (Nov. 13)**

I will close those lectures by discussing the probability distribution of the set of orbits in the general lattice random Lorentz gas. Those orbits are interesting objects which are self-avoiding and mutually avoiding lattice objects. The mathematical understanding of the properties of those random objects relevant to the derivation of Fick's law is at this stage close to zero. I will present numerical works and open questions.