CEMRACS 2017 - Seminar Session

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XVA analysis from the balance-sheet (joint work with Claudio Albanese)

Stéphane Crépey

LaMME, Université d'Évry

Friday 28/07 - 9am

Since the financial crisis, derivative dealers charge to their clients various add-ons, dubbed XVAs, meant to account for counterparty risk and its capital and funding implications. XVAs deeply affect the derivative pricing task by making it global, nonlinear, and entity dependent. However, before the technical implications, the fundamental points are to understand what deserves to be priced and what does not, and to establish, not only the pricing, but also the corresponding dividend and accounting policy of the bank.

Because banks cannot replicate jump-to-default related cash flows, deals trigger wealth transfers from bank shareholders to creditors and shareholders need to set capital at risk. On this basis, we devise a theory of XVAs, whereby so-called contra-liabilities and cost of capital are sourced from bank clients at trade inceptions, on top of the fair valuation of counterparty risk, in order to compensate shareholders for wealth transfer and risk on their capital.

The resulting all-inclusive XVA formula, to be sourced from clients incrementally at every new deal, reads CVA+FVA+KVA, where C sits for credit, F for funding, and where the KVA is a cost of capital risk premium. This formula is interpreted dynamically as the cost of the possibility for the bank to go into run-off, while staying in line with shareholder interest, from any point in time onward if wished.

Regress Later Monte Carlo for Optimal Control of Markov Chains

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A common approach in numerical solution of optimal control of discrete-time Markov processes is to discretise the process as a finite state space controlled Markov chain. This method suffers from the course of dimensionality and requires complex methods for design of efficient discretisation. On the contrary, Monte Carlo methods are not affected by the dimensionality of the state space. A successful adaptation of this approach to pricing of American option was performed in [4] and sparked a succession of papers. However, there it was crucial that the control (stopping) does not affect the dynamics of the underlying process. Adaptation of this approach to the problem of optimal control of discrete-time Markov processes was only achieved in [3], where the control was treated as an additional variable in the state space (control randomisation), but the convergence of the proposed scheme was not proved.

In this talk, I will present our approach in which we do not use control randomization. The key to our solution is the regress later approach developed to improve accuracy of American option pricing [2]. It consists of projecting the value function $V(t+1, x)$ over the linear space generated by basis functions $\{\phi_k(t+1, x)\}_{k=1}^{K}$ and then computing, possibly analytically,

$$\mathbb{E}[V(t+1, X_{t+1})|X_t = x, u_t = u] \approx \sum_{k=1}^{K} \alpha_k^{t+1} \mathbb{E}[\phi_k(t+1, X_{t+1})|X_t = x, u_t = u],$$

where u_t is the control at time t. The regress later approach allows us to: avoid randomisation of the control improving speed and accuracy; place training points freely leading to faster convergence; use performance iteration without resimulation saving computational time. It also allows us to prove that our numerical scheme converges, the task that seems very difficult for the control randomisation method [3].

In the talk I will introduce our approach, explain its links to existing solutions, sketch the convergence result and provide numerical evaluation. This talk extends our results in [1]. This is a joint work with Jan Palczewski.

References

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Project Evaluation under Uncertainty

Jorge P. Zubelli

IMPA

Monday 31/07 - 9am

Industrial strategic decisions have evolved tremendously in the last decades towards a higher degree of quantitative analysis. Such decisions require taking into account a large number of uncertain variables and volatile scenarios, much like financial market investments. Furthermore, they can be evaluated by comparing to portfolios of investments in financial assets such as in stocks, derivatives and commodity futures. This revolution led to the development of a new field of managerial science known as Real Options.

The use of Real Option techniques incorporates also the value of flexibility and gives a broader view of many business decisions that brings in techniques from quantitative finance and risk management. Such techniques are now part of the decision making process of many corporations and require a substantial amount of mathematical background. Yet, there has been substantial debate concerning the use of risk neutral pricing and hedging arguments to the context of project evaluation. We discuss some alternatives to risk neutral pricing that could be suitable to evaluation of projects in a realistic context with special attention to projects dependent on commodities and non-hedgeable uncertainties. More precisely, we make use of a variant of the hedged Monte-Carlo method of Potters, Bouchaud and Sestovic to tackle strategic decisions. Furthermore, we extend this to different investor risk profiles. This is joint work with Edgardo Brigatti, Felipe Macias, and Max O. de Souza.

If time allows we shall also discuss the situation when the historical data for the project evaluation is very limited and we can make use of certain symmetries of the problem to perform (with good estimates) a nonintrusive stratified resampling of the data. This is joint work with E. Gobet and G. Liu.

Splitting algorithm for nested events

Ludovic Goudenège

CNRS, Université Paris Est

Tuesday 01/08 - 9am

Consider a problem of Markovian trajectories of particles for which you are trying to estimate the probability of a event.

Under the assumption that you can represent this event as the last event of a nested sequence of events, it is possible to design a splitting algorithm to estimate the probability of the last event in an efficient way. Moreover you can obtain a sequence of trajectories which realize this particular event, giving access to statistical representation of quantities conditionally to realize the event.

In this talk I will present the "Adaptive Multilevel Splitting" algorithm and its application to various toy models. I will explain why it creates an unbiased estimator of a probability, and I will give results obtained from numerical simulations.

A limiting Hamilton-Jacobi equation arising in anomalous diffusion – memory effects

Álvaro MATEOS GONZÁLEZ, 420 S, UMPA ENS de Lyon, 46 Allée d'Italie, 69364 Lyon Cedex 07, FRANCE

Keywords : Structured PDE, Hamilton-Jacobi equations, Hyperbolic limit, Subdiffusion

The seminal articles [1] and [2] introduce Hamilton-Jacobi equations for the study of reaction-diffusion fronts. I wish to present a stability result that plays a preliminary role in the study of related phenomena in the context of subdiffusion. The biological motivation for this work is a model of random motion of cytoplasmic proteins subjected to trapping phenomena in crowded cellular media.
I will consider a linear age-structured partial differential equation describing a random walk in space coupled to waiting times distributed according to a heavy-tailed power law 1 studied in a joint work with Vincent Calvez and Pierre Gabriel [4]. I will present the convergence of the hyperbolic rescaling of that PDE towards a limiting Hamilton-Jacobi equation.

$$\begin{cases} \partial_t n(t,x,a) + \partial_a n(t,x,a) + \dfrac{\mu}{1+a} n(t,x,a) = 0 \\ n(t,x,0) = \displaystyle\int_0^\infty \int_{\mathbb{R}} \frac{\mu}{1+a} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-x'}{\sigma^2}\right)^2} n(t,x',a)\,\mathrm{d}x'\,\mathrm{d}a \\ n(0,x,a) = n^0(x,a) \qquad | \text{ supp } (n^0(x,\cdot)) \subset [0,1), \end{cases} \tag{1}$$

I will focus on two main complications encountered in the proof of the result, which are intimately linked to the subdiffusive nature of the studied dynamics, and on instructive aspects of the solutions we have found. The first is the non-existence of an integrable stationary measure, which we expected, since the solution of the homogeneous problem studied in [3] exhibits a self-similar decay. A time-corrected application of the maximum principle deals with this. The second complication is the long persistence in time of memory terms, which require a fine estimate of the contribution of the initial conditions to a renewal term. This appears in our viscosity supersolution proof.

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Projected particle methods for solving McKean-Vlasov stochastic differential equations

Denis Belomestny

Duisbuig-Essen University

Wednesday 02/08 - 9am

We propose a novel projection-based particle method for solving the McKean-Vlasov stochastic differential equations. Our approach is based on a projection-type estimation of the marginal density of the solution in each time step.

The projection-based particle method leads in many situation to a significant reduction of numerical complexity compared to the widely used kernel density estimation algorithms.

We derive strong convergence rates and rates of density estimation. The convergence analysis in the case of linearly growing coefficients turns out to be rather challenging and requires some new type of averaging technique.

This case is exemplified by explicit solutions to a class of McKean-Vlasov equations with affine drift.

The performance of the proposed algorithm is illustrated by several numerical examples.

Some asymptotic results for low dimensional optimal stopping - the problem of local volatility from American options

Stefano De Marco

CMAP, École Polytechnique

Wednesday 02/08 - 10am

The valuation of American options (a widespread type of financial contract) requires the numerical solution of an optimal stopping problem. Numerical methods for such problems have been widely investigated. Monte-Carlo methods are based on the implementation of dynamic programming principles coupled with regression techniques. In lower dimension, one can choose to tackle the related free boundary PDE with deterministic schemes.

Pricing of American options will therefore be inevitably heavier than the one of European options, which only requires the computation of a (linear) expectation. The calibration (fitting) of a stochastic model to market quotes for American options is therefore an a priori demanding task. Yet, often this cannot be avoided: on exchange markets one is typically provided only with market quotes for American options on single stocks (as opposed to large stock indexes - e.g. S&P500 - for which large amounts of liquid European options are typically available).

In this talk, we show how one can derive (approximate, but accurate enough) explicit formulas - therefore replacing other numerical methods, at least in a low-dimensional case - based on asymptotic calculus for diffusions.

More precisely: based on a suitable representation of the PDE free boundary, we derive an approximation of this boundary close to final time that refines the expansions known so far in the literature. Via the early premium formula, this allows to derive semi-closed expressions for the price of the American put/call. The final product is a calibration recipe of a Dupire's local volatility to American option data.

Based on joint work with Pierre Henry-Labordère.

An overview of contract theory with information asymmetry and extension to mean field models

Thibaut Mastrolia

CMAP, École Polytechnique

Thursday 03/08, 9am

In this talk, we investigate a situation in which two economical entities interact to maximize their own payoff with asymmetry information applied to contract theory.

One of them is called the Principal (she) and proposes to the second one, named the Agent (he), some incentives to modify her wealth. Basically, an employer (the Principal) aims at hiring an employee (the Agent) by giving to him a salary (the incentive) to manage a risky project.

The main difficulty encountered by the Principal comes from the fact that she observes the result of the work of her Agent without observing the work of him directly.

This kind of situation coincides exactly with a moral hazard problem in which the Principal has to design an employment contract given to her Agent to maximize her utility without observing directly his work. We identify this paradigm with a Stackelberg equilibrium that can be explicitly solved in several examples.

We begin to study the example of Holmstrom and Milgrom dealing with one Principal and one Agent in the continuous case.

We then extend this investigation to N-interacting Agents hired by one Principal by studying the impact of the strategy played by the Agents on the value of the Principal.

We finally focus on a model dealing with mean-field interactions between many Agents by solving explicit examples.

Mean field type control with congestion

Mathieu Laurière

NYU Shangaï

Wednesday 09/08, 9am

The theory of mean field type control (or control of MacKean-Vlasov) aims at describing the behaviour of a large number of agents using a common feedback control and interacting through some mean field term. The solution to this type of control problem can be seen as a collaborative optimum. We will present the system of partial differential equations (PDE) arising in this setting: a forward Fokker-Planck equation and a backward Hamilton-Jacobi-Bellman equation. They describe respectively the evolution of the distribution of the agents states and the evolution of the value function. Since it comes from a control problem, this PDE system differs in general from the one arising in mean field games.

Recently, this kind of model has been applied to crowd dynamics. More precisely, in this talk we will be interested in modeling congestion effects: the agents move but try to avoid very crowded regions. One way to take into account such effects is to let the cost of displacement increase in the regions where the density of agents is large. The cost may depend on the density in a non-local or in a local way. We will present one class of models for each case and study the associated PDE systems. The first one has classical solutions whereas the second one has weak solutions. Numerical results based on the Newton algorithm and the Augmented Lagrangian method will be presented. This is joint work with Yves Achdou.

On the discretization of some nonlinear Fokker-Planck-Kolmogorov equations and applications

Francisco Silva

Limoges University

Thursday 10/08, 9am

In this work, we consider the discretization of some nonlinear Fokker-Planck-Kolmogorov equations. The scheme we propose preserves the non-negativity of the solution, conserves the mass and, as the discretization parameters tend to zero, has limit measure-valued trajectories which are shown to solve the equation. This convergence result is proved by assuming only that the coefficients are continuous and satisfy a suitable linear growth property with respect to the space variable. In particular, under these assumptions, we obtain a new proof of existence of solutions for such equations.

We apply our results to several examples, including Mean Field Games systems and variations of the Hughes model for pedestrian dynamics.

VC results on Nonparametric Regression: beyond the i.i.d. case

David Barrera

CMAP, Ecole Polytechnique

Friady 11/08, 9am

In this talk, we will present a series of nonasymptotic results concerning the rate of convergence of the generalization error for least-squares regression estimates over a cloud of data not necessarily satisfying the i.i.d. hypothesis. These results are motivated, among others, by questions arising in MCMC methods and in Time Series Analysis.

In short, we will see that for clouds of data with bounded response (all the experiments occur within an "horizontal" strip), the i.d. (identically distributed) hypothesis is irrelevant for the arguments of the VC theory to go forward if one interprets the "goal" function as the best estimator of the response with respect to the average law of the data. Once these nonasymptotic rates are achieved, it is possible to use coupling to give rates on the generalization error under conditions on weak dependence.

These rates imply also the asymptotic consistency of the estimate, without additional assumptions, in a sense to be specified during the talk. In particular, we will see that the least squares regression method is well behaved and well controlled for geometrically (even: polynomially) ergodic Markov chains if one targets the regression function of the respective data with respect to its asymptotic distribution.

The results in this talk come from joint work with G.Fort and E.Gobet.

Laurent Mertz
NYU Shangaï
Wednesday 16/08, 9am

Title: Stochastic variational inequalities for random mechanics.

Abstract:
The mathematical framework of variational inequalities is a powerful tool to model problems arising in mechanics such as elasto-plasticity where the physical laws change when some state variables reach a certain threshold [1]. Somehow, it is not surprising that the models used in the literature for the hysteresis effect of non-linear elasto-plastic oscillators submitted to random vibrations [2] are equivalent to (finite dimensional) stochastic variational inequalities (SVIs) [3]. This presentation concerns (a) cycle properties of a SVI modeling an elasto-perfectly-plastic oscillator excited by a white noise together with an application to the risk of failure [4,5]. (b) a set of Backward Kolmogorov equations for computing means, moments and correlation [6]. (c) free boundary value problems and HJB equations for the control of SVIs. For engineering applications, it is related to the problem of critical excitation [7]. This point concerns what we are doing during the CEMRACS research project. (d) (if time permits) on-going research on the modeling of a moving plate on turbulent convection [8]. This is a mixture of joint works and / or discussions with, amongst others, A. Bensoussan, L. Borsoi, C. Feau, M. Huang, M. Laurière, G. Stadler, J. Wylie, J. Zhang and J.Q. Zhong.

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Existence to a calibrated regime-switching local volatility model

Alexandre Zhou

CERMICS, ENPC

Wednesday 16/08, 10am

By Gyongys theorem, a local and stochastic volatility (LSV) model is calibrated to the market prices of all European call options with positive maturities and strikes if its local volatility function is equal to the ratio of the Dupire local volatility function over the root conditional mean square of the stochastic volatility factor given the spot value. This leads to a SDE nonlinear in the sense of McKean. Particle methods based on a kernel approximation of the conditional expectation, as presented by Guyon and Henry-Labordre (2011), provide an efficient calibration procedure. But so far, no global existence result is available for the limiting SDE. We obtain existence in the special case of the LSV model called regime switching local volatility, where the stochastic volatility factor is a jump process taking finitely many values and with jump intensities depending on the spot level. This is a joint work with Benjamin Jourdain.

Forward and backward simulation of Euler scheme

Emmanuel Gobet

CMAP, Ecole Polytechnique

Thursday 17/08, 9am

We analyse how reverting Random Number Generator can be efficiently used to save memory in solving dynamic programming equation. For SDEs, it takes the form of forward and backward Euler scheme. Surprisingly the error induced by time reversion is of order 1.